Int. J. Heat Mass Transfer. Vol. 25, No. 5, pp. 643–651, 1982
Printed in Great Britain

0017-9310/82/050643-09 $03.00/0
Pergamon Press Ltd.

THERMAL CONDUCTIVITY OF METAL-FILLED SYSTEMS

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(*Received* 28 *April* 1981)

Abstract—An equation has been theoretically derived for the effective thermal conductivity of a medium with anisodiametric particles. The case of the thermal conductivity of particles being much in excess of that of the binding medium is analyzed in detail. The possibility is shown for the particle shape parameters to be determined from the dependence of the effective thermal conductivity on the filler volume fraction. The effective thermal conductivity of solutions filled with metallic powders differing in nature and dispersivity has been studied experimentally. The predicted results agree satisfactorily with the experimental data.

NOMENCLATURE

A_n,	depolarization factor;
a, b, c,	ellipsoid semi-axes;
c_e,	effective metal volume fraction;
c_i,	volume fraction of particles of *i*th kind;
$\mathbf{G}$,	temperature gradient in the medium far from a particle;
I_i,	first invariant of the particle polarizability tensor;
k,	particle shape factor;
N,	number of species of particles;
N_o,	number of measurements;
p,	significance level;
$\mathbf{p}_i$,	particle dipole moment;
$\mathbf{r}$,	position vector;
s,	r.m.s. variance;
S_i,	particle surface;
S_{oi},	surface bounding the volume V_{oi};
$t_p(N_o)$,	Student number;
T_b,	temperature of the binding medium;
T_d,	temperature of dispersed particles;
T_r,	temperature of reference medium;
$\Delta T_r, \Delta T_{st}$,	probe temperature variation in reference and studied medium, respectively;
V,	volume of the system containing sufficiently large number of particles;
V_f,	bulk powder volume;
V_i,	particle volume;
V_{oi},	volume of the system containing the *i*th particle.

Greek symbols

$\alpha_{kn}^{(i)}$,	particle polarizability tensor components;
λ_b,	thermal conductivity of binding medium;
λ_d,	thermal conductivity of dispersed particles;
λ_e,	effective thermal conductivity of the system;
λ_f,	thermal conductivity of filled solution;
λ_r,	thermal conductivity of reference medium;
λ_s,	thermal conductivity of solution;
λ_{st},	thermal conductivity of the medium studied;
ρ_f,	bulk powder density;
ρ_m,	metal density;
ρ_s,	solution density.

1. INTRODUCTION

COMPOSITIONS based on resins or other viscous media with fillers (highly dispersed metal powders) are being widely used in different branches of engineering. The objective of this study was to determine the dependence of the effective thermal conductivity of such suspensions upon the content of the filler whose thermal conductivity differs substantially from that of the medium. The results obtained can be used for both estimation of the thermal conductivity of metallic powders from their composition and preparation of suspensions with a specified thermal conductivity. The relationship between the thermal conductivity of a suspension and its composition can also be used to determine the filler content in suspension.

The dispersion medium of the composition studied was a viscous solution of polyisobutylene in kerosene which precluded filler sedimentation. The fillers were highly dispersed metal powders differing in dispersivity and the shape of particles. In concentrated solutions of polymers in hydrocarbon solvents filled with metallic powders, sedimentation is extremely slow due to a high viscosity of the solution. Therefore the composition can be considered uniform and isotropic. An increase in the volume concentration of metal leads to formation by metallic particles of a branched net-like structure actually throughout the entire composition volume. In this case the dispersed system can be considered conveniently as consisting of two mutally penetrating components: solution and metal. The properties of such systems are treated in detail in [1].

At relatively moderate volume concentrations of the filler the composition consists of the binding medium (solution) and randomly distributed metallic inclusions [1]. Although the thermal conductivity of metal surpasses that of the solution by several orders of magnitude, the effective thermal conductivity of the system, at small concentrations of metallic powder, depends solely on the nature of the filler, i.e. volume content, dispersivity etc., but not on the nature of the metal itself. Usually the powder particles are covered with an oxide film, the thermal conductivity of which is $1\frac{1}{2}$ or 2 orders of magnitude higher than that of the binding medium. The screening effect of the envelope is small and may be neglected, as shown in [2]. For the same reason, variation of the heat conduction coefficient in the boundary layer of the polymer adjacent to the particle may also be ignored.

In liquid polymer compositions the powder particles may form flocculi consisting of several particles—the phenomenon which is conditioned by the aggregation process. The nature of formation and the size of flocculi have been studied in a number of works [3–5]. It should be noted that formation of aggregates is accompanied not only by a change in the anisodiametric nature of inclusions, but by manifestation of the following factor: the volume fraction of particles in the aggregate exceeds the mean fraction of dispersed phase. It has been shown in [6] that this factor is important only in the case when the dispersed phase volume concentration exceeds 25%.

2. CALCULATION OF THE EFFECTIVE THERMAL CONDUCTIVITY OF SUSPENSION

In order to calculate the effective thermal conductivity of a liquid polymer medium with a metallic filler, consider a homogeneous system with the thermal conductivity λ_b containing randomly oriented anisodiametric inclusions with the thermal conductivity $\lambda_d \gg \lambda_b$. The effective thermal conductivity of similar systems has been determined in a number of works [7–9] for a wide range of the filler volume concentrations by isolating a unit cell from the system and making calculations for it.

There is also a statistical method for the solution of this problem. The effective thermal conductivity of composition is then determined from

$$\lambda_e \langle \nabla T \rangle = \lambda_d \langle \nabla T_d \rangle + \lambda_b \langle \nabla T_b \rangle$$

and

$$\langle \nabla T \rangle = \langle \nabla T_d \rangle + \langle \nabla T_b \rangle. \tag{1}$$

Here T_b and T_d are the temperature fields in the medium and in inclusions, the angled brackets denote averages over the system volume. The temperature fields are found from the solution of the Laplace equation

$$\nabla^2 T_d = \nabla^2 T_b = 0 \tag{2}$$

subject to the boundary conditions

$$T_b|_{\text{surf}} = T_d|_{\text{surf}}; \quad \lambda_b \nabla T_b \cdot \mathbf{n}|_{\text{surf}} = \lambda_d \nabla T_d \cdot \mathbf{n}|_{\text{surf}}, \tag{3}$$

where $\mathbf{n}$ is a unit vector normal to the surface.

Let us rearrange equation (1) by means of the formulae

$$\langle \nabla T_d \rangle = \frac{1}{V} \sum_{(i)} \int_{V_i} \nabla T_d \, dv = \frac{1}{V} \sum_{(i)} \oint_{S_i} T_d \, d\mathbf{S},$$

$$\langle \nabla T_b \rangle = \frac{1}{V} \int_{V - \sum_{(i)} V_i} \nabla T_b \, dv = \frac{1}{V} \left[\sum_{(i)} \left(\oint_{S_{oi}} T_b \, d\mathbf{S} - \oint_{S_i} T_b \, d\mathbf{S} \right) + \int_{V - \sum_{(i)} V_{oi}} \nabla T_b \, dv \right],$$

where V is the composition volume, V_{oi} the volume containing the *i*th particle only, S_{oi} the surface bounding this volume, V_i, S_i is the volume and surface of the *i*th particle, respectively.

Equations (2) and (3) yield

$$\oint_{S_{oi}} [\mathbf{r}(\nabla T_b \cdot d\mathbf{S}) - T_b \, d\mathbf{S}] = \frac{\lambda_d - \lambda_b}{\lambda_b} \oint_{S_i} T_d \, d\mathbf{S}$$

where $\mathbf{r}$ is the position vector. Then, with account for the above expressions we shall obtain for (1)

$$(\lambda_e - \lambda_b) \left(\int_{V - \sum_{(i)} V_{oi}} \nabla T_b \, dV + \sum_{(i)} \oint_{S_{oi}} T_b \, d\mathbf{S} \right) = -\lambda_b \sum_{(i)} \oint_{S_{oi}} [T_b \, d\mathbf{S} - \mathbf{r}(\nabla T \, d\mathbf{S})]. \tag{4}$$

When the content of particles is small, then, to evaluate the integrals over the surfaces S_{oi} at a distance from the particle, we shall restrict ourselves to the dipole expansion of the field T_b

$$T_b = -\mathbf{G} \cdot \mathbf{r} + \frac{\mathbf{p}_i \cdot \mathbf{r}}{r^3}, \tag{5}$$

where $\mathbf{p}_i$ is the dipole moment of the inclusion determined from equations (2) and (3) subject to the boundary condition

$$\nabla T_b|_{r \to \infty} \to -\mathbf{G}.$$

Since the thermal problem is linear, then

$$\mathbf{p}_i = V_i \sum_{k,n=1}^{3} \alpha_{kn}^{(i)} \mathbf{e}_k, \tag{6}$$

where $\alpha_{kn}^{(i)}$ is the particle polarizability tensor [10], $\mathbf{e}_k$ are the unit vectors.

Taking into account the random orientation of particles

$$\langle \cos^2 \alpha \rangle = \langle \cos^2 \beta \rangle = \langle \cos^2 \gamma \rangle = \frac{1}{3},$$

where α, β, γ are the angles between the principal axes of the inclusion polarizability tensor and the temperature gradient, will yield

$$\frac{\lambda_e - \lambda_b}{\lambda_e + 2\lambda_b} = \sum_{j=1}^{N} \frac{4}{9}\pi I_j c_j. \qquad (7)$$

Here, the summation is taken over the N types of inclusions, c_j is the volume concentration of the jth inclusion,

$$I_j = \sum_{k=1}^{3} \alpha_{kk}^{(j)}$$

is the first invariant of the tensor α_{kk}: $I_j > 0$ at $\lambda_d > \lambda_b$; $I_j < 0$ at $\lambda_d < \lambda_b$. When $\lambda_d \gg \lambda_b$ and $\lambda_d \ll \lambda_b$, the value of $|I_j|$ is independent of λ_d and is determined only by the shape of inclusions. At $N = 1$, equation (7) is reduced to the Maxwell-type relation

$$\frac{\lambda_e - \lambda_b}{\lambda_e + 2\lambda_b} = kc, \quad k = \frac{4}{9}\pi I, \qquad (8)$$

in which c is the volume content of particles.

The coefficient k allows for non-sphericity of a particle. Equation (8) holds also for inclusions of the same nature but of different shape. In this case the averaged polarizability tensor invariant is equal to

$$I = \sum_{j=1}^{N} p_j I_j,$$

where p_j are the fractions of particles of dissimilar shapes. For particles of ellipsoidal shape the polarizability tensor is presented in [10]

$$k = \frac{1}{9}\sum_{n=1}^{3} \frac{\lambda_d - \lambda_b}{\lambda_b + (\lambda_d - \lambda_b)A_n}, \qquad (9)$$

where A_n are the depolarization factors. For spheres with $A_n = \frac{1}{3}$ the well known Maxwell formula is obtained:

$$k = \frac{\lambda_d - \lambda_b}{\lambda_d + 2\lambda_b}.$$

Equation (8) with the coefficient (9) coincides with a similar equation for dielectric penetration obtained in [11, 12] where the mutual effect of particles is accounted for by the Lorentz formula for interaction of induced point dipoles. The technique of deriving equation (8) used in the present work makes it possible to avoid application of the comparison method employed by Polder, Van Santen and Fricke. In the derivation of equation (8) it is assumed that the centres of inclusions are uniformly distributed throughout the composition and, therefore, the form of the correlation function for mutual arrangement of particles is not taken into account. Buevich and co-workers [13] have developed the methods for calculation of the effective characteristics of composition—viscosity and thermal conductivity—with regard for the binary function of inclusions. Numerical calculations made for spherical particles [14] have shown that the effective thermal conductivity of composition at $c \lesssim 0.3$ is weakly dependent upon the form of the binary distribution function.

Based on Fricke's equation [15] for a dispersed medium with ellipsoidal inclusions,

$$\lambda_e - \lambda_b = \frac{c}{3}\sum_{n=1}^{3} \frac{\lambda_d - \lambda_b}{\lambda_b + (\lambda_d - \lambda_b)A_n}[\lambda_b(1 - A_n) - \lambda_e A_n], \qquad (10)$$

Hamilton [16] has suggested an empirical relationship for thermal conductivity of the medium with inclusions

$$\frac{\lambda_e - \lambda_b}{\lambda_e + (n-1)\lambda_b} = c \cdot \frac{\lambda_d - \lambda_b}{\lambda_d + (n-1)\lambda_b} \qquad (11)$$

where n is the empirical coefficient accounting for the anisodiametricity of inclusions. An equation similar to (11) is used to calculate the electrical conductivity of liquid disperson systems [17], with n determined from experimental data. For $\lambda_d \ll \lambda_b$ and $\lambda_d \gg \lambda_b$ we have

$$\frac{\lambda_e - \lambda_b}{\lambda_e + (n-1)\lambda_b} = -\frac{c}{n-1}, \quad \lambda_d \ll \lambda_b;$$

$$\frac{\lambda_e - \lambda_b}{\lambda_e + (n-1)\lambda_b} = c, \quad \lambda_d \gg \lambda_b. \qquad (12)$$

Comparison of equations (8) and (12) with the Maxwell formula shows:

$$\frac{\lambda_e - \lambda_b}{\lambda_e + 2\lambda_b} = c, \quad \lambda_d \gg \lambda_b$$

$$\frac{\lambda_e - \lambda_b}{\lambda_e + 2\lambda_b} = -\frac{c}{2}, \quad \lambda_d \ll \lambda_b. \qquad (13)$$

It is seen from (8) and (13) that in (8) anisodiametricity leads to a change in the effective volume fraction of anisodiametric particles by k times at $\lambda_d \gg \lambda_b$ and by $2k$ times at $\lambda_d \ll \lambda_b$ as compared with similar volume content of spherical inclusions. In equation (12), in contrast to (8), an increase in the effective thermal conductivity of the composition involving anisodiametric particles, as compared with the same content of spherical particles, is associated with a $(n-1)/2$-fold increases in the effective thermal conductivity of the medium at $\lambda_d \gg \lambda_b$, while at $\lambda_d \ll \lambda_b$—also with an $(n-1)/2$-fold decrease of the effective filler volume. Physically this interpretation is incorrect.

In the case of randomly oriented inclusions their shape can be modelled by an effective axisymmetric ellipsoid. The coefficient k is determined by equation (8) from the experimental data. Then, with the use of equation (9), one can determine the relationship between the ellipsoid semi-axes. The depolarization factors A_n can be calculated from [10]:

$$A_1 = \frac{1 - e^2}{2e^3}\left(\ln\frac{1 + e}{1 - e} - 2e\right),$$

$$e = \sqrt{\left(1 - \frac{b^2}{a^2}\right)}, \quad a > b = c;$$

$$A_1 = \frac{1+e^2}{e^3}(e - \operatorname{arctg} e),$$

$$e = \sqrt{\left(\frac{b^2}{a^2} - 1\right)},$$

$$b = c > a. \tag{14}$$

Here b, a, c are the ellipsoid semi-axes, $A_1 = (a - A_1)/2$, $A_3 = A_2$.

Figure 1 shows the dependence of the coefficient k upon the ellipsoid shape for different relationships between thermal conductivities of the dispersed, λ_d, and binding, λ_b, media.

The study of the effect of the shape of inclusions on polarizability has been carried out in [17]. The experiments have shown that polarizability is determined, in the main, by the relationship between the length of the particle axes rather than the particle shape.

The author of [18] has suggested that the relationship between the semi-axes of an equivalent ellipsoid be determined by measuring dielectric permittivity of the dispersion system.

3. ANALYSIS OF EXPERIMENTAL DATA

Thermal conductivity of filled polymer compositions has been measured by the constant-power probe method [20–22] in view of certain features of the system under study: high viscosity, poor wettability, slow structure formation processes in the medium. The probe in the form of a thin needle was immersed into the medium. After turn-on of the heater a change of the probe temperature in time has been recorded. The measurements are made over a quasi-stationary heating length. A relative version of this method is used when the rates of heating of two identical probes immersed into the studied and reference media are compared. Identical electric power is supplied to the probes. The ratio between the thermal conductivities of the medium studied λ_{st} and of the reference medium λ_r is determined from

$$\frac{\lambda_{st}}{\lambda_r} = \frac{\Delta T_r}{\Delta T_{st}} \tag{15}$$

where ΔT_r and ΔT_{st} are temperature changes in the reference and studied media. For optimization of the measurement technique the thermal conductivities of hydrocarbon liquids thickened with 5.7 and 15% of polyisobuthelene (PIB) have been determined. The reference medium was a pure solvent. The confidence interval for the thermal conductivity λ was determined [19] from

$$\bar{\lambda} - t_p(N_o - 1)\frac{s}{\sqrt{N_o}} < \lambda < \bar{\lambda} + t_p(N_o - 1)\frac{s}{\sqrt{N_o}},$$

$$s^2 = \frac{1}{\sqrt{(N_o - 1)}}\sum_{i=1}^{N_o}(\lambda_i - \bar{\lambda})^2,$$

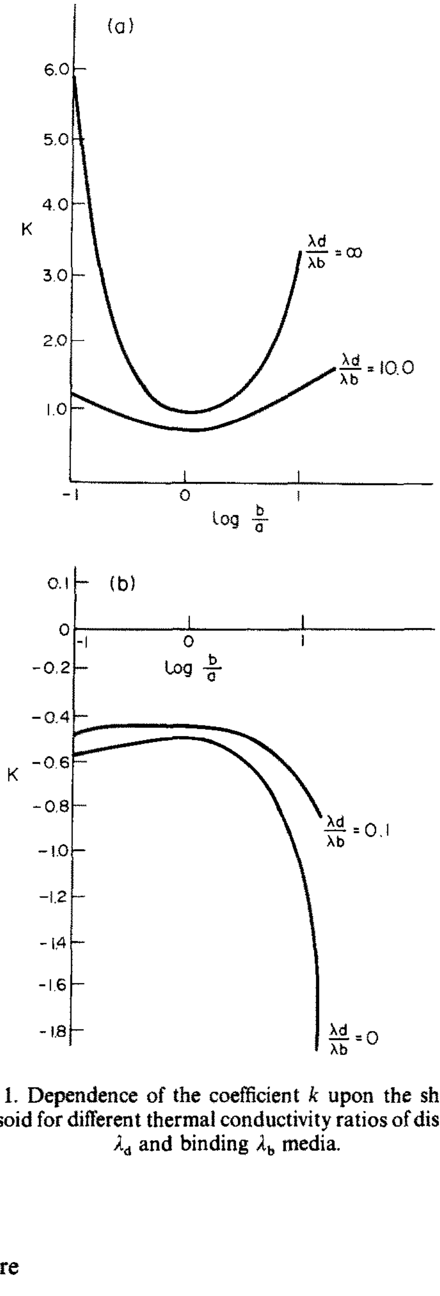

FIG. 1. Dependence of the coefficient k upon the shape of ellipsoid for different thermal conductivity ratios of dispersed λ_d and binding λ_b media.

where

$$\bar{\lambda} = \left(\sum_{i=1}^{N_o}\lambda_i\right)\Bigg/N_o, \quad t_p(N_o)$$

is the Student number for the significance level $p = 0.05$ for N_o measurements, s is the r.m.s. variance. The analysis of the preliminary experiments and statistical treatment of the results obtained allowed the development of the technique of thermal conductivity measurement. The measurements have shown that the thermal conductivity of the solution is actually independent of the PIB content (c = 5%, $\bar{\lambda}_s$ = 0.114 Wt/MK, c = 7%, $\bar{\lambda}_s$ = 0.112 Wt/MK, c = 15%, $\bar{\lambda}_s$ = 0.112 Wt/MK) which is in accord with the reported data proving near coincidence between the thermal conductivities of pure solvents and PIB [27, 28].

Table 1. Characteristics of powders

No.	$\rho_m \cdot 10^{-3}$ kg/m^3	$\rho_f \cdot 10^{-3}$ kg/m^3	ρ_m/ρ_f	V_f(cm^3) min	V_f(cm^3) max	c min	c max	c_e min	c_e max
1	1.74	0.58	3.28	12.8	89.8	0.037	0.324	0.049	0.453
2a	2.15	0.69	3.12	13.5	80.9	0.042	0.350	0.121	0.394
b		1.10	1.95	10.0	60.0	0.049	0.491	0.065	0.448
3a	2.74	0.75	3.65	12.6	25.2	0.056	0.126	0.034	0.095
b		0.80	3.43	18.8	61.5	0.056	0.245	0.013	0.242
c		0.95	2.88	15.8	53.0	0.056	0.254	0.034	0.258
d		1.18	2.32	10.0	50.2	0.042	0.328	0.026	0.284
4	2.74	0.18	15.22	10.0	276.0	0.006	0.244	0.088	0.670
5	5.25	0.55	9.54	10.0	72.7	0.009	0.102	0.016	0.097

The above technique has been used to measure the thermal conductivity of solutions filled with metallic powders. The solutions were hydrocarbon liquids, with close thermal conductivities, into which 10% of PIB had been added. The powders were : (1) milled magnesium; (2) two fractions of the aluminum–magnesium alloy; (3) four fractions of spherical aluminum; (4) aluminum powder, and (5) gamma-iron oxide (Table 1). The granulometric composition of the powders is presented in Fig. 2. The densities of solutions are almost the same and equal to $\rho_s = 900\,\mathrm{kg/m^3}$. The measurements were carried out for different bulk volumes V_f of powders per 0.1 kg of composition. The volume content of metal is found from

$$c = \frac{(\rho_f/\rho_m)\,V_f}{(\rho_f/\rho_m)V_f + (0.1 - \rho_f V_f)/\rho_s}. \tag{16}$$

Here ρ_f is the bulk density of powder and ρ_m is the

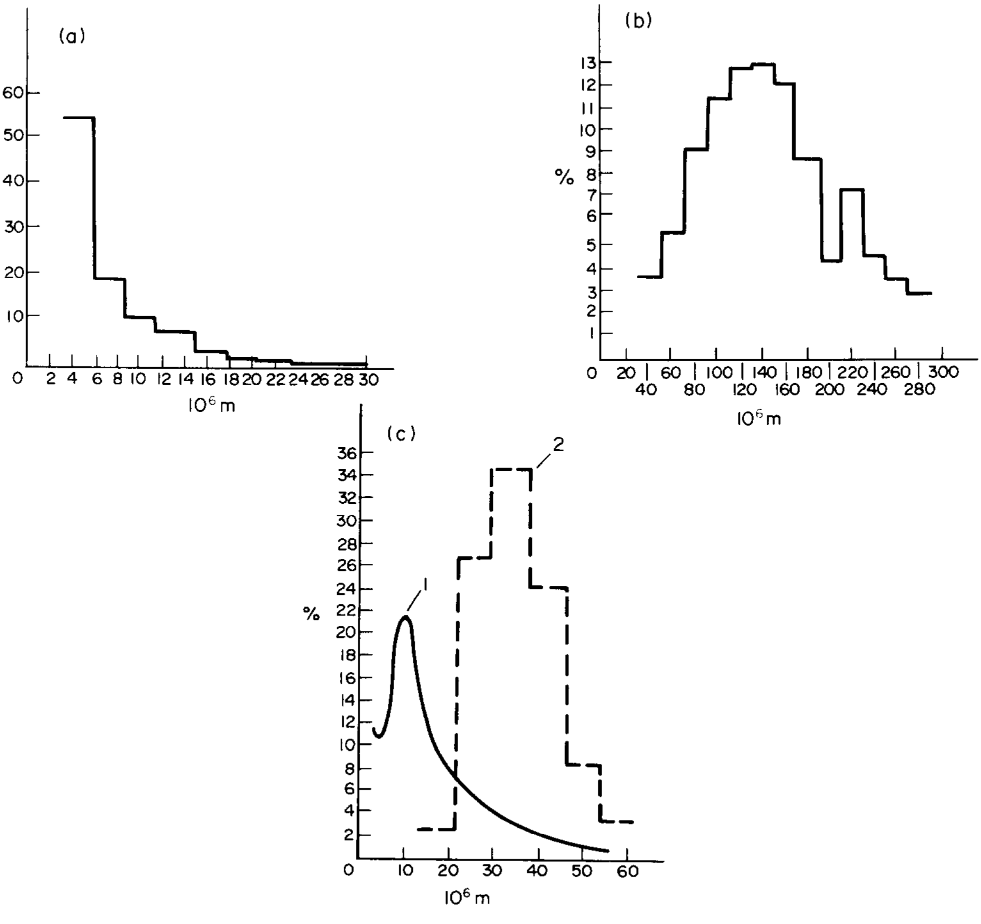

FIG. 2. Granulometric composition of powders: (a) magnesium; (b) magnesium with aluminum; (c) aluminum powder (solid line); 2, spherical aluminum.

metal density. The effective volume content of metal is calculated from the experimental data by

$$c_e = \frac{(\lambda_f/\lambda_s) - 1}{(\lambda_f/\lambda_s) + 2} \qquad (17)$$

where λ_f/λ_s is the ratio of thermal conductivities of the filled, λ_f, and pure, λ_s, solutions. The value of c_e is numerically equal to the volume fraction of well conducting particles ($\lambda_d/\lambda_s \gg 1$) the addition of which results, according to (8), in the value of λ_f/λ_s determined experimentally.

Table 1 presents the densities ρ_f, ρ_m and ranges of values V_f, c, c_e observed in experiments. The relationship between the quantities c and c_e is described by the regressive equation

$$c_e = n + mc. \qquad (18)$$

In view of the fact that the Maxwell formula is applicable to spherical particles at small volume contents of the dispersed phase, the linear equation (18) for aluminum powder was obtained from the data for $c_e \lesssim 0.35$. Over the whole range of experimental data the quantities c_e and c are related through

$$c_e = 0.049 + 4.568c - 8.284c^2.$$

Further analysis of the significance of coefficients n, m [19] in equation (18) has shown that the free term n can be neglected, i.e. an equation of the type (8) can be obtained

$$c_e = kc. \qquad (19)$$

Figure 3 shows the curves of (19), experimental points and the values of the coefficient k for magnesium, aluminum and gamma-iron oxide.

All of the above compositions consist of two components: solution and powder of one metal. We have analyzed the possibility for calculation of the effective thermal conductivity of the system consisting of three components: solution, aluminum powder and gamma-iron oxide powder. The thermal conductivity has been calculated by equation (17) at

$$c_e = c_\gamma k_\gamma + c_{Al}k_{Al}, \qquad (20)$$

where c_γ, c_{Al} are the volume contents of the gamma-iron oxide and aluminum powder, k_γ and k_{Al} are their coefficients. The results of calculations prove the applicability of this equation (see Table 2). The experiments carried out confirm the applicability of equations (17) and (20) for determination of the effective thermal conductivity of polymer solutions filled with metallic powders at volume contents $c_e \lesssim$

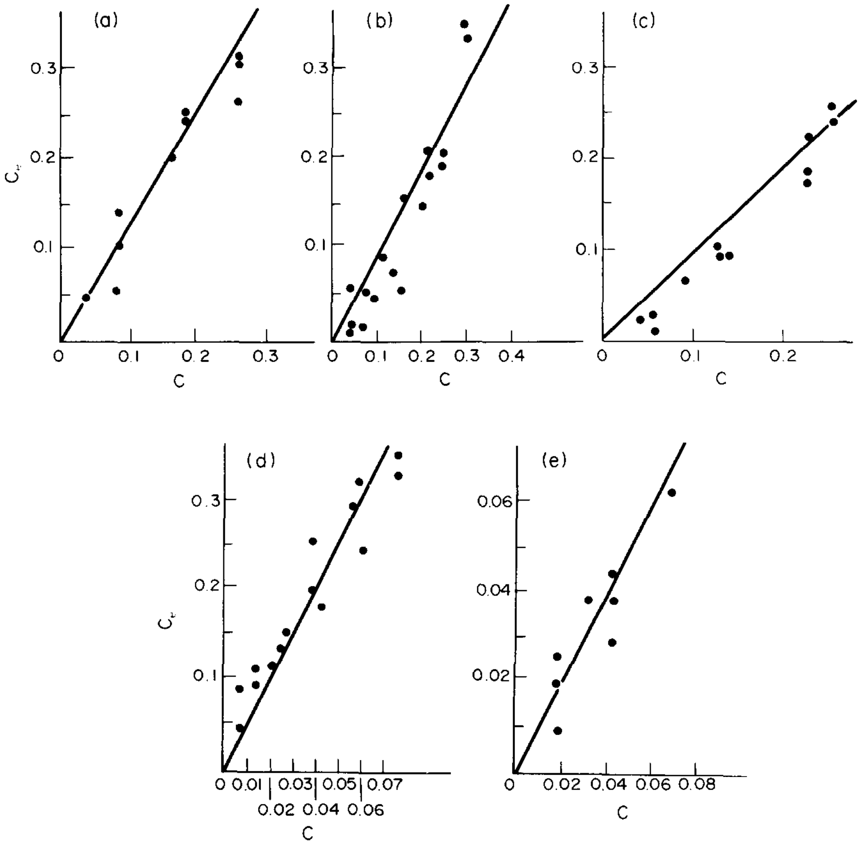

FIG. 3. Dependence of c_e upon c: (a) magnesium, $k \approx 1.294$; (b) aluminum–magnesium, $k \approx 0.952$; (c) spherical aluminum, $k \approx 0.960$; (d) aluminum powder, $k \approx 5.268$; (e) gamma-iron oxide, $k \approx 1.000$.

Table 2. Three-component system ($k_{Fe_\gamma} = 1.00, k_{Al} = 4.11$)

No.	c_{Fe_γ}	c_{Al}	c_e^{exp}	c_e^{pred}
1	0.0101	0.0127	0.0566	0.0630
2	0.0213	0.0134	0.0567	0.0776
3	0.0332	0.0069	0.0446	0.0637
4	0.0337	0.0141	0.0991	0.0938
5	0.0342	0.0214	0.1525	0.1245
6	0.0468	0.0073	0.0566	0.0799
7	0.0475	0.0149	0.1045	0.1118
8	0.0483	0.0227	0.1573	0.1446
9	0.0630	0.0158	0.1279	0.1320
10	0.1641	0.0241	0.1620	0.1670

0.35. In the case of powders 2, 3, 5, with particles close in shape to the spherical one, $k \approx 1$.

Figure 4 shows the photographs of the powder particles of magnesium, aluminum–magnesium alloy, first fraction of spherical aluminum and aluminum powder. The magnesium particles are of distinct anisodiametric nature (Fig. 4a). The coefficient k calculated from the experimental data is equal here to 1.29. Figures 4 (b) and (c) show the powder particles of the aluminum–magnesium alloy and of spherical aluminum. Here the coefficients k are close to unity ($k_{Al+Mg} \approx 0.952$, $k_{Al} \approx 0.960$). The particles of aluminum powder combine into chains resulting in substantial anisodiametricity of the flocculi formed ($k = 5.298$).

Equation (8) has been used for the analysis of experimental data reported in [20–23] where the effective thermal conductivities of rubbers filled with powders of non-spherical particles were measured. The thermal conductivities of fillers were taken from [24, 25]. Table 3 represents the basic experimental data from [20–23] with the values of k calculated from these data by the present authors. Based on the values of k, the parameters of the effective ellipsoid have been calculated: the coefficient A_1 along the rotation axis and the ratio between its long and short semi-axes. The results of calculations for model fillers show that the semi-axes ratio provides an acceptable estimate for the anisodiametricity degree of particles. The experimental studies carried out confirm the applicability of equation (8) for determination of the thermal conductivity of compositions with anisodiametric particles at relatively small filler volume concentrations. The semi-axes ratio of an effective rotation ellipsoid, calculated from the experimental data, provides an adequate qualitative characteristic of the anisodiametricity degree.

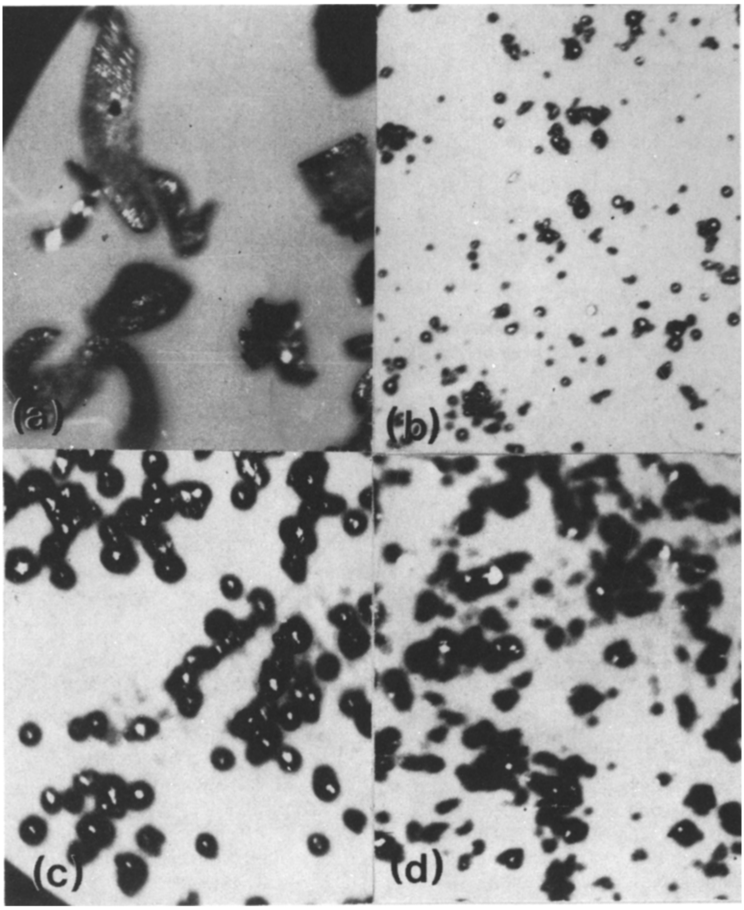

Fig. 4. Photographs of the particles of powders: (a) magnesium; (b) aluminum–magnesium alloy; (c) spherical aluminum; (d) aluminum powder.

Table 3. Results of experimental data processing based on thermal conductivity of polymer compositions

Filler	λ_d (Wt/MK)	Shape of inclusions	Size	c (%)	λ_s, (Wt/MK) T, K	λ_f/λ_s	k	A_1^*	b/a^*
[23] Copper powder	394.3 [26]	non-spherical	50 and	10.0	0.08	1.750	1.8052	0.0842	3.68
			15	20.0	20	2.625		0.7311	4.55
				10.0	0.22	1.9545	2.1256	0.0673	4.35
				20.0	300	4.545		0.7758	5.71
[24] Copper powder	394.3 [26]	non-spherical	13	13.0	0.076	1.974	1.5353	0.1071	3.03
				24.5	20	2.632		0.6758	3.54
[25] Industrial diamond	629 [27]	non-spherical	0.5–1.0	4.8	0.21	1.429	2.0971	0.0686	4.29
		arbitrary		15.2	300	2.381		0.7724	5.60
[25] Crystallized quartz	10.42 [27]	irregular	11	10.3	0.21	1.429	1.1433	0.1522	2.25
				21.4	300	1.952		0.5768	2.40
[25] Corundum	28.5 [27]	plates	6.9	8.0		1.619	2.0915	0.0688	4.29
				17.0		2.095		0.7718	5.59
				7.2		1.476	1.8463	0.0734	4.06
			1.8	11.6		1.905		0.7535	5.09
				21.3					
[16] Balm wood	0.045 [27]	disks	800 ×	14.0	0.22	0.756	−0.5182	0.8995	14.3
			7200	25.0	300	0.677			
[16] Aluminum	207 [27]	parallelepipedes	1600 ×			1.986	1.5887	0.1016	3.17
			1600 ×	15.5				0.6886	3.75
			400						
		cylinders	2700 ×	15.5		2.165	2.3750	0.0584	4.85
			270					0.8013	6.60
			5000 ×	15.5		1.872	1.4524	0.1171	2.82
			1000					0.6534	3.23
								0.1436	2.36
Magnesium	165 [27]	milled	Data of the present work				1.2935	0.5989	2.60
Aluminum powder	207 [27]	chains					5.2683	0.0232	9.17
									16.9

* Upper figures correspond to an elongated ellipsoid, the lower—to a prolated one.

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CONDUCTIVITE THERMIQUE DES SYSTEMES A REMPLISSAGE METALLIQUE

Résumé—Une équation a été établie théoriquement pour la conductivité thermique effective d'un milieu avec des particules de diamètre varié. On analyse en détail le cas de particules étant en excès par rapport au milieu de liaison. On montre la possibilité pour les paramètres de forme des particules de les déterminer à partir de la dépendance de la conductivité thermique effective vis-à-vis de la fraction volumique du matériau de remplissage. A été étudiée expérimentalement la conductivité thermique effective de solutions remplies de poudres métalliques qui diffèrent en nature et en dispersion. Les calculs s'accordent avec les résultats expérimentaux.

DIE WÄRMELEITFÄHIGKEIT VON MEDIEN MIT METALLFÜLLUNG

Zusammenfassung—Es wurde theoretische eine Beziehung für die effektive Wärmeleitfähigkeit eines Mediums abgeleitet, das Partikel mit ungleichen Durchmessern enthält. Der Fall, bei dem die Wärmeleitfähigkeit der Partikel sehr groß gegenüber derjenigen des umschließenden Mediums ist, wird ausführlich behandelt. Es wird gezeigt, wie man die Formparameter der Partikel aus der Abhängigkeit der effektiven Wärmeleitfähigkeit vom Volumenverhältnis des Füllstoffes bestimmen kann. Die effektive Wärmeleitfähigkeit von Lösungen, die metallische Pulver unterschiedlicher Art und Dispersionsfähigkeit enthielten, wurde experimentell untersucht. Die theoretisch berechneten Werte stimmen zufriedenstellend mit den Versuchsergebnissen überein.

ТЕПЛОПРОВОДНОСТЬ МЕТАЛЛОНАПОЛНЕННЫХ СИСТЕМ

Аннотация — Приведен теоретический вывод формулы для эффективной теплопроводности среды с анизодиаметрическими частицами. Подробно проанализирован случай, когда теплопроводность частиц значительно превышает теплопроводность связующей среды. Показана возможность определения параметров формы частицы по зависимости эффективной теплопроводности от объемной доли наполнителя.

Проведено экспериментальное исследование эффективной теплопроводности систем, наполненных металлическим порошком разной природы и дисперсности. Результаты расчетов удовлетворительно согласуются с экспериментальными данными.